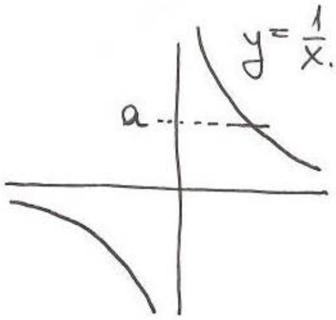
Exame Fu. Ma. Xan.'17

a) $\left|\frac{1}{x}-\frac{1}{3}\right| \geq \frac{1}{x} \iff \begin{cases} \frac{1}{x}-\frac{1}{3} \geq \frac{1}{x} & \text{se } \frac{1}{x}-\frac{1}{3} \geq 0 \\ \frac{1}{x}-\frac{1}{3} \leq -\frac{1}{x} & \text{se } \frac{1}{x}-\frac{1}{3} < 0 \end{cases}$

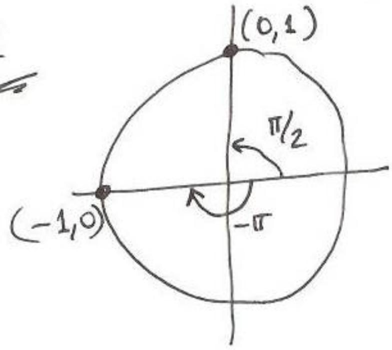


$\begin{cases} \frac{1}{3} \leq 0 \text{ se } \frac{1}{x} \geq \frac{1}{3};\ x \in \emptyset \cap (0,3] = \emptyset \\ \frac{1}{x} \leq \frac{1}{6} \text{ se } \frac{1}{x} < \frac{1}{3};\ x \in (-\infty, 0) \cup [6, +\infty) \end{cases}$

$\frac{1}{x} \geq a \iff x \in (0, a]$

$\frac{1}{x} \leq a \iff x \in (-\infty, 0) \cup [a, +\infty)$

b) $\frac{\operatorname{sen}(x-\pi)}{\cos(x+\pi/2)} = \frac{\operatorname{sen} x \cos(-\pi) + \cos x \operatorname{sen}(-\pi)}{\cos x \cos\frac{\pi}{2} - \operatorname{sen} x \operatorname{sen}\frac{\pi}{2}} = \frac{-\operatorname{sen} x}{-\operatorname{sen} x} = 1$



c) $2x^2+7x+3=0$; $x = \frac{-7 \pm \sqrt{49-24}}{4} = \begin{cases} -1/2 \\ -3 \end{cases}$

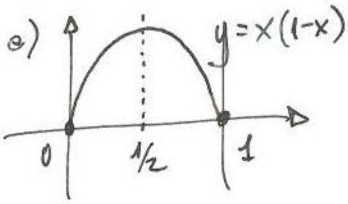
$2x^2+7x+3 = 2\left(x+\frac{1}{2}\right)(x+3)$

$\lim_{x\to -3} \frac{x^2-9}{2x^2+7x+3} = \lim_{x\to -3} \frac{(x-3)(x+3)}{2\left(x+\frac{1}{2}\right)(x+3)} = \lim_{x\to -3} \frac{x-3}{2\left(x+\frac{1}{2}\right)} = \frac{-6}{2\left(-\frac{5}{2}\right)} = \frac{6}{5}.$

d) Derivando, $2yy' = 1 + \frac{x}{y}\,\frac{y'\cdot x - y}{x^2}.$

En $(1,1)$, $2\cdot y' = 1 + \frac{y'-1}{1}$; $y'=0$

e)



$y = x(1-x)$ é unha parábola con máximo en $x=\frac{1}{2}$ con valor $\frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{1}{4}$.

Como $y=\tan(x)$ é derivable en $[0,\frac{1}{4}]$, $\tan(x(1-x))-1$ é derivable en $[0,1]$. Ademáis $\tan(x(1-x))$ é idéntica en $x=0$ e $x=1$.

Logo existe $c\in(0,1)$ tal que $f'(c)=0$.

f) $l_1(x) = \dfrac{x-2}{1-2} \cdot \dfrac{x-4}{1-4} = \dfrac{1}{3}(x^2-6x+8)$; $\log_2(1) = 0$

$l_2(x) = \dfrac{x-1}{2-1} \cdot \dfrac{x-4}{2-4} = -\dfrac{1}{2}(x^2-5x+4)$; $\log_2(2) = 1$.

$l_3(x) = \dfrac{x-1}{4-1} \cdot \dfrac{x-2}{4-2} = \dfrac{1}{6}(x^2-3x+2)$; $\log_2(4) = 2$

$$\phi(x) = -\frac{1}{2}(x^2-5x+4) + \frac{1}{3}(x^2-3x+2) = \frac{1}{6}(-3x^2+15x-12+2x^2-6x+4) =$$

$$= \frac{1}{6}(-x^2+9x-8) = -\frac{x^2}{6} + \frac{3}{2}x - \frac{4}{3}$$

Ejercicio

a) $y = \dfrac{1}{1-x} = (1-x)^{-1}$; $y(0) = 1$.

$y' = -(1-x)^{-2}(-1) = (1-x)^{-2}$; $y'(0) = 1$.

$y'' = (-2)(1-x)^{-3}(-1) = (1-x)^{-3}$; $y''(0) = 2$

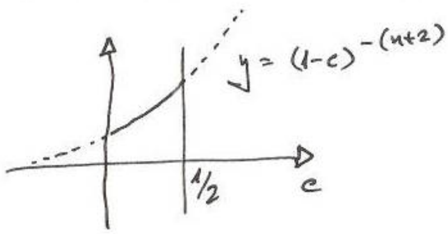
$y''' = 2 \cdot 3 (1-x)^{-4}$; $y'''(0) = 6$

...

$y^{(n)} = n!\,(1-x)^{-(n+1)}$; $y^{(n)}(0) = n!$

b) $$\left|y\left(\tfrac{1}{2}\right) - T_n\left(\tfrac{1}{2}\right)\right| = \left|R_{n+1}\left(\tfrac{1}{2}\right)\right| = \left|\frac{y^{(n)}(c)}{(n+1)!}\left(\tfrac{1}{2}\right)^{n+1}\right| = \left|\frac{(n+1)!\,(1-c)^{-(n+2)}}{(n+1)!}\left(\tfrac{1}{2}\right)^{n+1}\right|$$

$$\max_{c \in [0,\frac{1}{2}]} \left|(1-c)^{-(n+2)}\right| = \left(\tfrac{1}{2}\right)^{-(n+2)}$$



En resumen,

$$\left|y\left(\tfrac{1}{2}\right) - T_n\left(\tfrac{1}{2}\right)\right| \leq \left|\frac{(n+1)!\left(\frac{1}{2}\right)^{-(n+2)}}{(n+1)!}\left(\tfrac{1}{2}\right)^{n+1}\right| = \frac{\left(\frac{1}{2}\right)^{n+1}}{\left(\frac{1}{2}\right)^{n+2}} = \frac{1}{\frac{1}{2}} = 2.$$